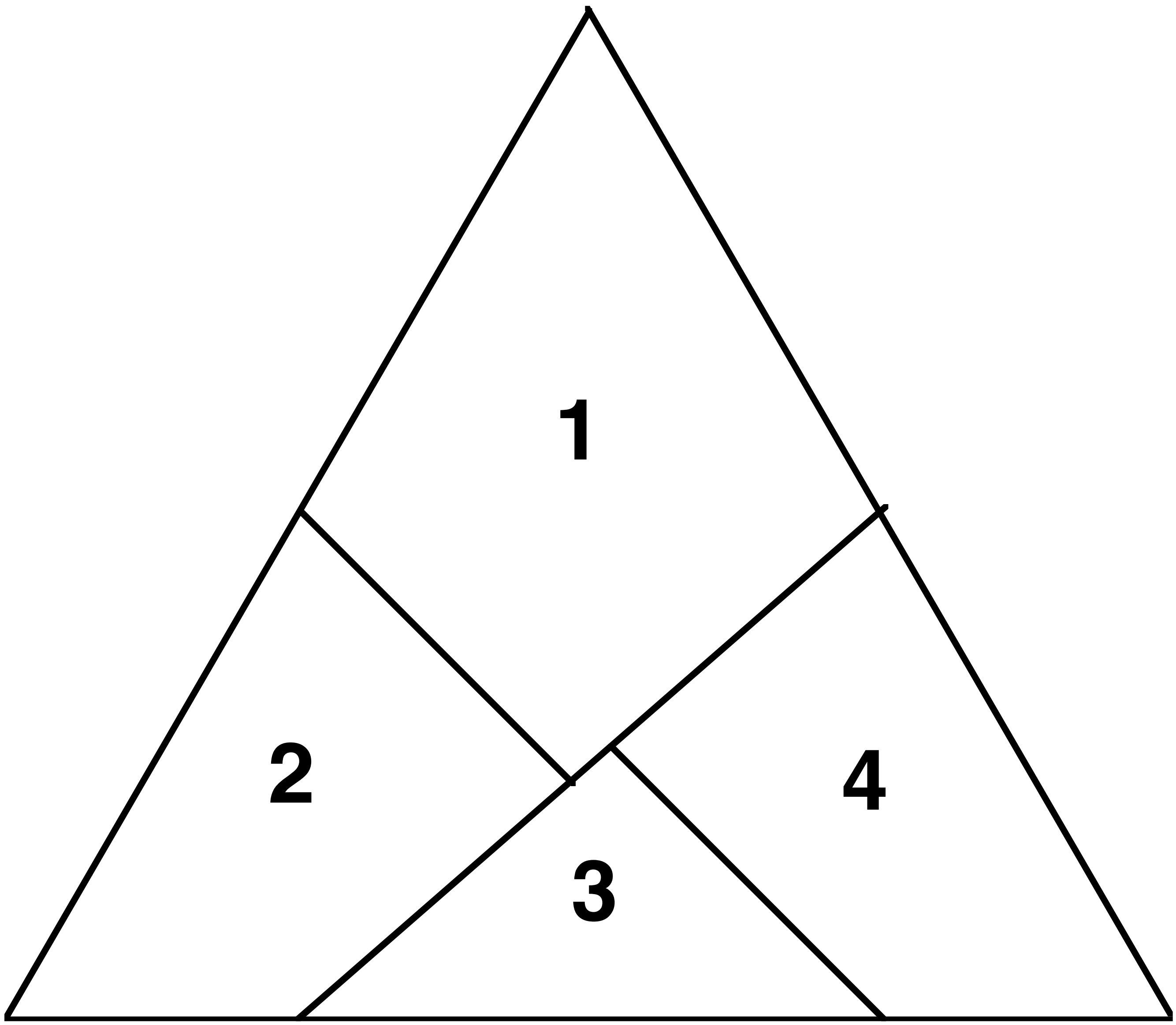
The Haberdasher's Puzzle

Equilateral Triangle to Square

Cut out the 4 pieces of the Equilateral Triangle shown below.

Rearrange them to form a square



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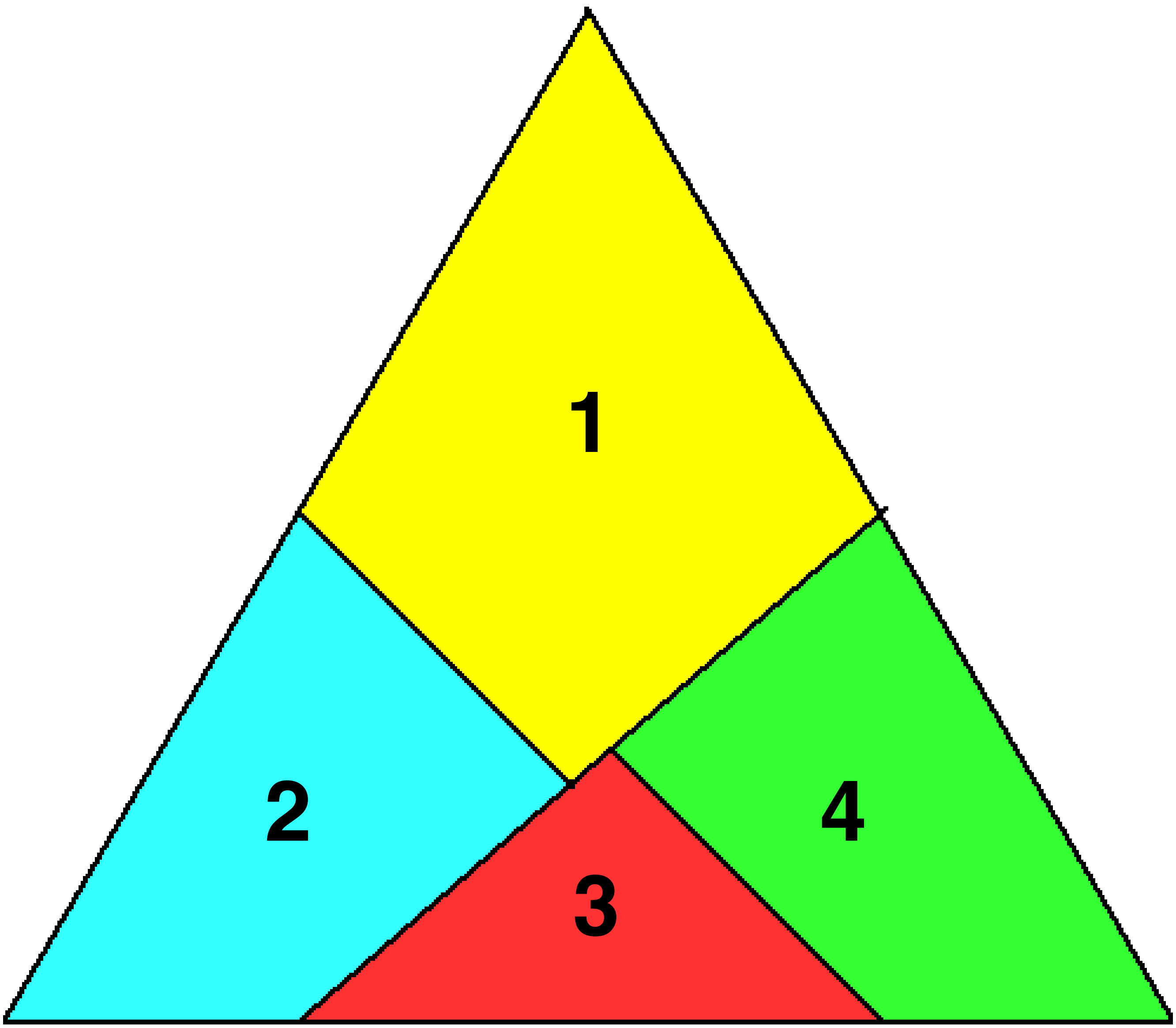

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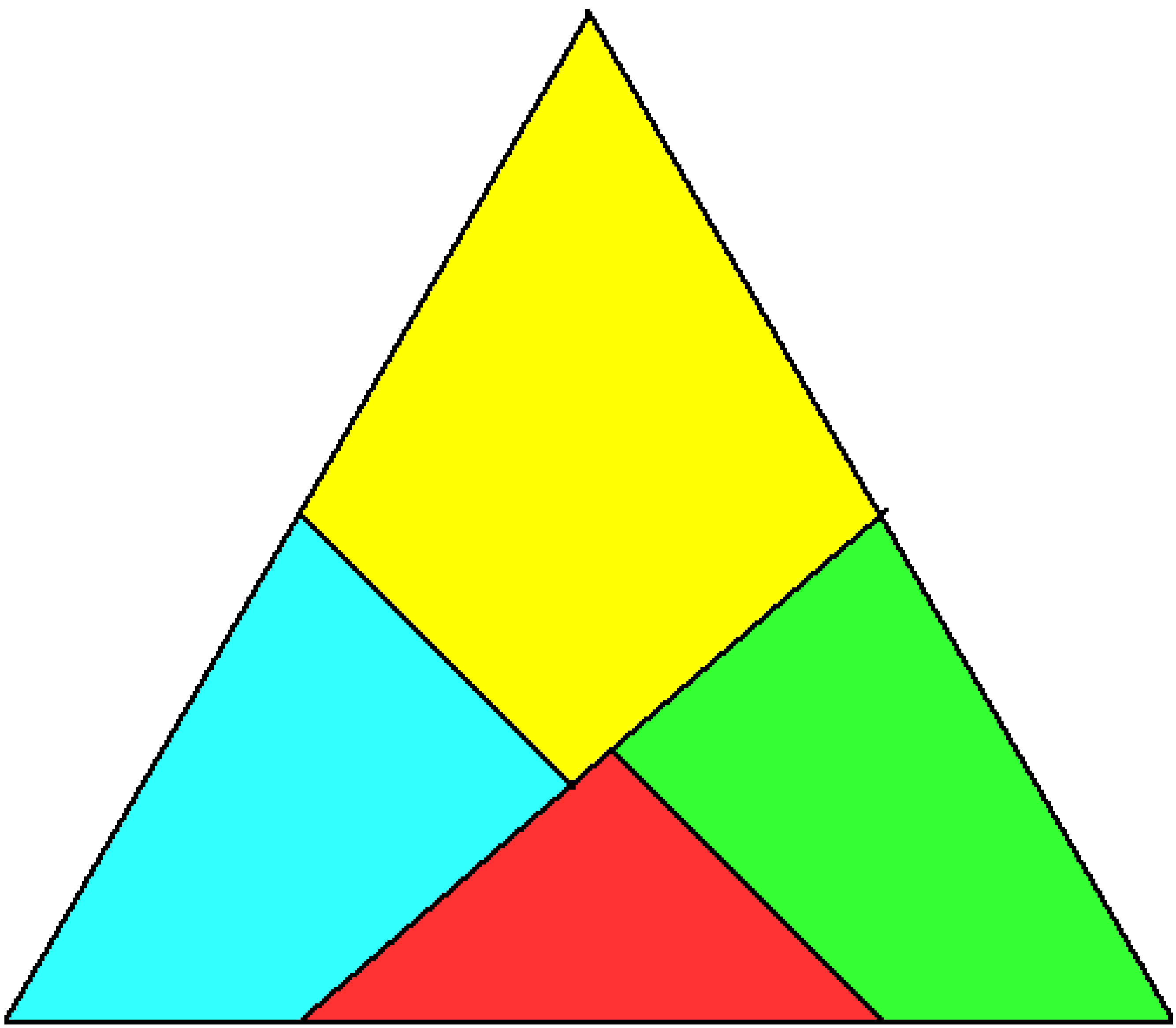
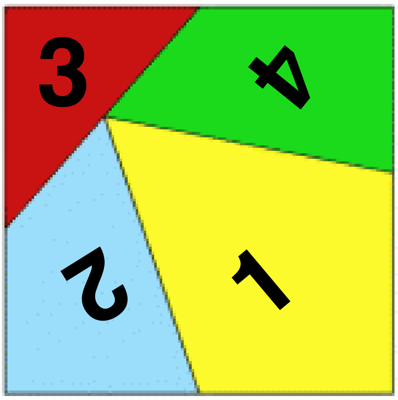
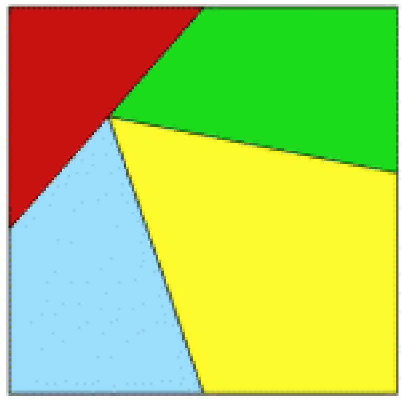


Solution:





Henry Ernest Dudeney

Henry Ernest Dudeney (10 April 1857–23 April 1930) was an English author and mathematician who specialized in logic puzzles and mathematical games. He is known as one of his country's foremost creators of puzzles. His last name is pronounced with a long "u" and a strong accent on the first syllable, as in "scrutiny".

Dudeney's first puzzle contributions were submissions to newspapers and magazines, often under the pseudonym of "Sphinx." Much of this earlier work was a collaboration with American puzzlist Sam Loyd.

Two of his most famous innovations was his 1903 success at solving the Haberdasher's Puzzle (Cut an equilateral triangle into four pieces that can be rearranged to make a square) and publishing the first known crossnumber puzzle, in 1926. In addition, he has been credited with inventing verbal arithmetic and discovering new applications of digital roots. (Wikipedia)

The Haberdasher's Puzzle, the greatest mathematical discovery of Henry Dudeney, was first published by him in the Weekly Dispatch in 1902 and then as problem no. 26 in The Canterbury Puzzles (1907).

A remarkable feature of the solution is that the each of the pieces can be hinged at one vertex, forming a chain that can be folded into the square or the original triangle. Two of the hinges bisect sides of the triangle, while the third hinge and the corner of the large piece on the base cut the base in the approximate ratio 0.982: 2: 1.018. Dudeney showed just such a model of the solution, made of polished mahogany with brass hinges, at a meeting of the Royal Society on May 17, 1905.

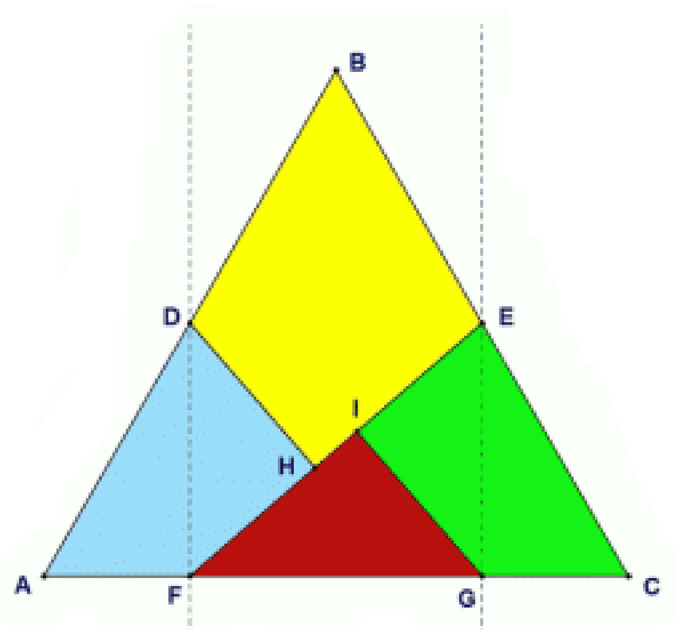
This problem comes from the book Canterbury Puzzles by Henry Ernest Dudeney. It os puzzle # 26. A FREE copy of this book as an e-book can be downloaded from gutenburg,org

Constructions:

Two different ways to construct the puzzle are given below. These are the instructions and drawings Dudeney used in his notes about the problem.

Construction using bisections and perpendiculars:

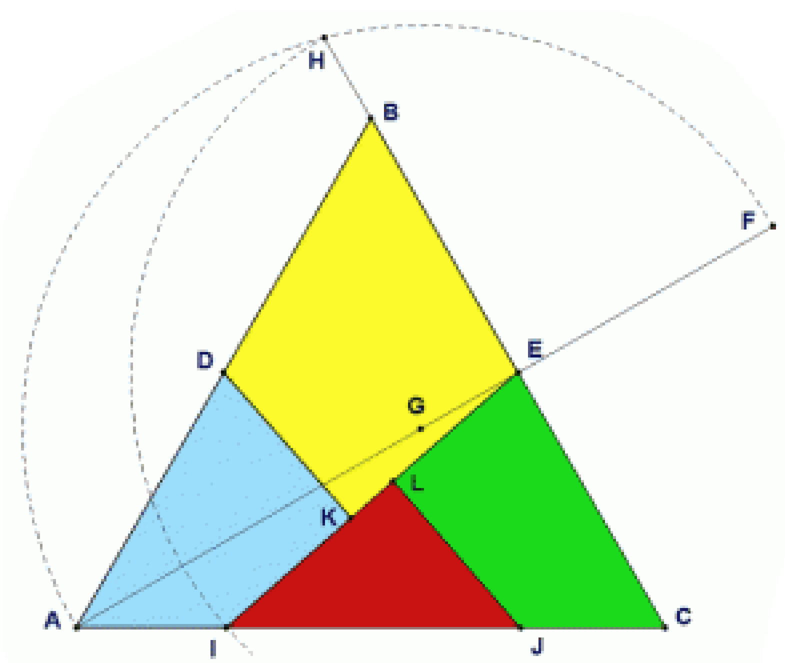
1. Draw an equilateral triangle with points A, B and C
2. Bisect AB at D and BC at E.
3. Draw the perpendicular lines from D and E on the side AC.
4. Determine the intersection points F and G on the side AC
5. Draw the line EF.
6. Draw the perpendiculars from D and G to this segment and determine the intersection points H and I.



Construction using arcs:

This is an elegant construction but much more difficult for students:

1. Draw an equilateral triangle with points A, B and C
2. Bisect AB at D and BC at E.
3. Extend AE to F so that EF = EB.
4. Bisect AF to create point G
5 Using point G as the center, draw the arc AHF.
6. Extend EB to H.
7. With point E as the center, draw the arc HI.
8. Make IJ = BE
9. Draw line IE
10. From D and J draw perpendiculars on IE to obtain points K and L.



Teachers Notes:

Why to you use numbers?

I have given you 3 different puzzles printouts. One black and white and one colored that are numbered and a colored puzzle that is not numbered. The reason for this lies in the problem of reflection. Some puzzles can only be solved if all the pieces face up. If some of the pieces were facing up and some were facing down the puzzle may not be possible to complete. Some shapes stay the same when flipped over (reflected) like the square or rectangle. Some have a “right or left” orientation to them when reflected. Parts 4 and 2 are like this. Part 2 cannot be rotated so that it matches part 2. It can be flipped over to match part 2. The parts must be reflections of each other to solve the puzzle. If one was flipped over it would cause a problem.

For this reason, I normally number the parts. The numbers allow you to be sure all the sides are facing up. They also make it easy to refer to a part. I often ask students to look at a specific numbered part tell me what you notice.

Practice vocabulary:

Most of the puzzles I post can be used in upper elementary and junior high. They also could be used in a high school or college geometry class but the use would differ.

In an upper elementary class I would present the puzzle and have them try to solve it. I might use it as a sponge activity or I might give it to them at the end off class to do at lunch (sneaky) When I have them in class together, I would then have a student come up and show the solution. That is where most puzzles end. For this reasons teachers may not use puzzles because they are trying to focus on core material.

A master teacher knows that puzzles are interesting to students and that they can be used to support core material if you use them correctly.

I would ask the students to look at the puzzle in the shape of the equilateral triangle. See the rhombus that is not a square (kite). See the right triangle. Is it isosceles. Are any sides cut in half. We call the bisected. Are any segments (part of a line) parallel to other segments? Are any perpendicular? If the parts form a square where are the 4 right angles in the triangle found? See how much **vocabulary** you can **practice** with this simple puzzle.

Are the area of the square and the triangle the same? How can you tell? Rearranging parts of rectangles but maintaining the area is a common way to introduce factoring in beginning algebra classes so this is a great question to ask. If they know the area formulas they can measure the shapes and use the numbers to compute the area.

See the 2 quadrilaterals, parts 4 and 5. Are they identical in shape? If so they can be placed one on the other and match up. I avoid the phrase “lie on top of each other” , after all its’ not biology. If you “reflect” part 4 does it now match up with part 2? Parts 3 and 4 can make a triangle but parts 2 and 4 cannot. This is due to the reflective difference between the parts. These shapes are called anti morphemic. Chemistry classes look at 3-D molecules that act this way. Organic chemistry will use this idea. Certain cancer drugs display this behavior. I know you are not a chemistry teacher, but students need to see these advanced ideas introduced somewhere and the 5’th grade is a great place.

Why would a high school teacher or college teacher use this puzzle?

Teachers in either level geometry course may use it to practice vocabulary but they would then take the problem to another level. The two construction methods to design the puzzle rely on the construction methods they are using in class. Each method has unique features so both are of interest. The puzzle is the motivation to do the construction.

They may also explore the relationship between the sides of the triangle and sides of the square.

The area of an equalaterial triangle with a side of S_T is

$$A_T = \frac{(S_T)^2 \bullet \sqrt{3}}{4}$$

The area of a Square is with a side of S_S is

$$A_S = (S_S)^2$$

The area of our equalaterial triangle = The area of our Square

so $A_T = A_S$

$$\frac{(S_T)^2 \bullet \sqrt{3}}{4} = (S_S)^2$$

$$(S_T)^2 = \frac{(S_S)^2 \bullet 4}{\sqrt{3}}$$

$$\sqrt{(S_T)^2} = \sqrt{\frac{(S_S)^2 \bullet 4}{\sqrt{3}}}$$

$$S_T = \frac{2 \bullet (S_S)}{\sqrt[4]{3}}$$

$S_T \cong 1.520 \bullet S_S$ or $S_S \cong .658 \bullet S_T$

The square made from the equilateral triangle will have a side about 65.8 % as long as the side of the triangle. You can measure the sides of the equilateral triangle and the sides of the square and find out if this is true for their figure.

The side of the square is twice the length of the leg of the right isosceles triangle in the original figure. Can you show this with the puzzle parts?